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International Journal of Multiphase Flow 32 (2006) 850–863

International Journal of Multiphase Flow

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Estimating gas holdup via pressure difference measurements in a cocurrent bubble column

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Received 6 May 2005; received in revised form 6 February 2006

Abstract

Estimating gas holdup via pressure difference measurements is a simple and low-cost non-invasive technique to study gas holdup in bubble columns. It is usually used in a manner where the wall shear stress effect is neglected, termed Method II in this paper. In cocurrent bubble columns, when the liquid velocity is high or the fluid is highly viscous, wall shear stress may be significant and Method II may result in substantial error. Directly including the wall shear stress term in the determination of gas holdup (Method I) requires knowledge of two-phase wall shear stress models and usually requires the solution of non-linear equations. A new gas holdup estimation method (Method III) via differential pressure measurements for cocurrent bubble columns is proposed in this paper. This method considers the wall shear stress influences on gas holdup values without calculating the wall shear stress. A detailed analysis shows that Method III always results in a smaller gas holdup error than Method II, and in many cases, the error is significantly smaller than that of Method II. The applicability of Method III in measuring gas holdup in a cocurrent air–water–fiber bubble column is examined. Analysis based on experimental data shows that with Method III, accurate gas holdup measurements can be obtained, while measurement error is significant when Method II is used for some operational conditions.

Keywords: Bubble column; Gas holdup; Gas–liquid flow; Measurement error; Pressure difference; Wall shear stress

1. Introduction

Gas holdup is defined as the fraction occupied by the gas phase in the total volume of a two- or three-phase mixture in a bubble column. It is one of the most important parameters characterizing bubble column hydrodynamics because it not only gives the volume fraction of the gas phase, it is also needed to estimate the interfacial area and thus the mass transfer rate between the gas and liquid phases (Shah et al., 1982).

Gas holdup can be measured by numerous invasive or non-invasive techniques, which have been reviewed by Kumar et al. (1997) and Boyer et al. (2002). Included among these techniques is a widely used method to estimate gas holdup via pressure difference measurements. This method has been used in semi-batch bubble

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doi:10.1016/j.ijmultiphaseflow.2006.02.008

columns (Ueyama et al., 1989; Luo et al., 1997; Zahradnik et al., 1997; Lin et al., 1998; Letzel et al., 1999; Su and Heindel, 2003, 2004), as well as airlift reactors (Hills, 1976; Merchuk and Stein, 1981; Al-Masry, 2001) and cocurrent bubble columns (Kara et al., 1982; Kelkar et al., 1983; Tang and Heindel, 2004, 2005a,b), where there is a net upward liquid flow. With this method, gas holdup is measured using the time-averaged static pressure drop along the column. The resulting gas holdup is an average value (both temporal and spatial) over the volume of the dispersion between the corresponding pressure taps. In semi-batch bubble column operations, Kara et al. (1982) and Tang (2005) showed that the gas holdup values obtained via the pressure difference method matched well (within $\pm 3\%$) with those obtained via direct gas holdup measurement (i.e., estimating gas holdup by measuring the mixture or liquid level before and after dynamic gas disengagement).

In applying the pressure difference method, manometers were initially installed along multiphase flow columns to measure pressure signals (Hills, 1976; Merchuk and Stein, 1981; Kara et al., 1982; Kelkar et al., 1983; Zahradnik et al., 1997; Al-Masry, 2001). Recently, pressure transducers have been used (Ueyama et al., 1989; Luo et al., 1997; Lin et al., 1998; Letzel et al., 1999; Su and Heindel, 2003, 2004; Tang and Heindel, 2004, 2005a,b), and they are usually flush mounted to the column wall so that the disturbance to the flow caused by the pressure transducers is minimized.

Measuring gas holdup via a pressure difference is simple and non-invasive and does not interrupt bubble column operation. With the price drop of piezoelectric pressure transducers and the development of computer data acquisition technology, this method becomes a convenient low-cost gas holdup measurement technique and is applicable to systems at high temperature and pressure (Luo et al., 1997; Lin et al., 1998; Letzel et al., 1999). This technique does not require a transparent fluid or containment vessel, nor does it have requirements on liquid electrical properties. It can be used to measure the overall average gas holdup in a multiphase column, as well as the average gas holdup in a column section. Thus, it can be used to probe the axial gas holdup variation in a column (Hol and Heindel, 2005). Compared to radiation attenuation methods (e.g., γ-ray or X-ray tomography), the pressure difference method is much safer. Furthermore, in addition to estimating gas holdup, pressure signals can also be used to determine flow regime transition (Vial et al., 2000; Ruthiya et al., 2005) and average bubble size (Chilekar et al., 2005) in bubble columns. When a solid phase is present, the pressure difference method can be used to measure gas holdup if the liquid–solid slurry behaves as a pseudo-homogeneous mixture or if the solid concentration as a function of height is known (Kumar et al., 1997).

Assuming one-dimensional isothermal flow, steady-state, constant cross-section, negligible mass transfer between the gas and liquid phases, and constant properties in a cross-section, Merchuk and Stein (1981) used a separated flow model of Wallis (1969) for vertical gas–liquid cocurrent flows to determine gas holdup in gas–liquid bubble columns and airlift reactors:

$$\varepsilon = \left(1 + \frac{1}{\rho_l g}\frac{dp}{dz}\right) + \frac{4\tau_w}{\rho_l D_c g} + \frac{U_l^2}{g}\frac{1}{(1-\varepsilon)^2}\frac{d\varepsilon}{dz} \tag{1}$$

where ε and p are the local gas holdup and pressure at position z, respectively, ρ_l is the liquid density, g is the acceleration due to gravity, D_c is the column inner diameter, U_l is the superficial liquid velocity, and τ_w is the wall shear stress. Hills (1976) obtained a similar expression assuming a pseudo-homogeneous two-phase mixture.

The first term on the right hand side of Eq. (1) accounts for the hydrostatic head, the second term describes wall shear effects, and the third term represents fluid acceleration due to void changes. The contribution of the acceleration term is typically $\sim 1\%$ of the total gas holdup (Merchuk and Stein, 1981). Hills (1976) has shown that in the worst case in a study with superficial liquid and gas velocities as high as 2.7 m/s and 3.5 m/s, respectively, the acceleration term amounted to less than 10% of the total gas holdup. As a result, the acceleration term is usually neglected in practice (Hills, 1976; Merchuk and Stein, 1981; Zahradnik et al., 1997; Al-Masry, 2001; Tang and Heindel, 2004). Without the acceleration term, Eq. (1) becomes

$$\varepsilon = \left(1 + \frac{1}{\rho_l g}\frac{dp}{dz}\right) + \frac{4\tau_w}{\rho_l D_c g} \tag{2}$$

To obtain the average gas holdup $\bar{\varepsilon}$ in a column section between two locations separated by a distance $\Delta z = z_2 - z_1$ (>0), average both sides of Eq. (2) from z_1 to z_2:

$$\frac{1}{\Delta z}\int_{z_1}^{z_2} \varepsilon\,\mathrm{d}z = \frac{1}{\Delta z}\int_{z_1}^{z_2}\left(1+\frac{1}{\rho_l g}\frac{\mathrm{d}p}{\mathrm{d}z}\right)\mathrm{d}z + \frac{1}{\Delta z}\int_{z_1}^{z_2}\frac{4\tau_w}{\rho_l D_c g}\,\mathrm{d}z \tag{3}$$

Thus

$$\bar{\varepsilon}_I = \bar{\varepsilon} = \left(1-\frac{1}{\rho_l g}\frac{\Delta p}{\Delta z}\right) + \frac{4\bar{\tau}_w}{\rho_l D_c g} \tag{4}$$

where $\Delta p = p_1 - p_2$ (>0) with p_1 and p_2 the pressures at location z_1 and z_2, respectively, and $\bar{\tau}_w$ represents the average wall shear stress in the same column section. The gas holdup measurement based on Eq. (4) is denoted *Method I* in the following discussion and the gas holdup value obtained with Method I is represented by $\bar{\varepsilon}_I$; it totally accounts for the wall shear stress effects and provides accurate gas holdup values based on the assumptions above.

The wall shear term in Eq. (4) is usually neglected for semi-batch bubble columns (Ueyama et al., 1989; Zahradnik et al., 1997; Su and Heindel, 2003, 2004). For cocurrent bubble columns and airlift reactors, this term is small at low superficial liquid velocities (e.g., $U_l \sim 1$ cm/s in air water systems). When the wall shear term is negligible, Eq. (4) can be simplified to

$$\bar{\varepsilon}_{II} = 1 - \frac{1}{\rho_l g}\frac{\Delta p}{\Delta z} \tag{5}$$

The gas holdup measurement based on Eq. (5) is called *Method II* in the following discussion and $\bar{\varepsilon}_{II}$ denotes the gas holdup value obtained with Method II. This method totally neglects the effects of wall shear stress.

The wall shear term in Eq. (4) increases significantly with increasing superficial liquid (U_l) and gas (U_g) velocities and can amount to ~20% of the total gas holdup (Hills, 1976; Merchuk and Stein, 1981). This is because the wall shear stress $\bar{\tau}_w$ increases significantly with U_l and U_g (Wallis, 1969; Liu, 1997; Magaud et al., 2001). When the liquid phase is highly viscous, the wall shear term can be significant even at superficial liquid velocities on the order of ~2–10 cm/s (Al-Masry, 2001). Hence, it is necessary to include the wall shear effect in the total gas holdup value for most cocurrent or viscous flow cases.

To calculate the wall shear term in Eq. (4) requires estimation of the two-phase wall shear stress $\bar{\tau}_w$, which is a complex function of gas holdup, superficial gas and liquid velocity, liquid phase rheological properties, and wall roughness. The models for $\bar{\tau}_w$ in gas–liquid two-phase flows are limited, and most are not general and cannot be extended beyond their restricted conditions (Gharat and Joshi, 1992). The two-phase wall shear stress is even more difficult to estimate when the liquid phase is non-Newtonian (Al-Masry, 2001). Even when a model for $\bar{\tau}_w$ is known, the model is usually a highly non-linear function of gas holdup (Herringe and Davis, 1978; Merchuk and Stein, 1981; Metkin and Sokolov, 1982; Beyerlein et al., 1985), and one has to solve a non-linear version of Eq. (4) to obtain the gas holdup. This is inconvenient, especially when a large number of data points are acquired.

In this paper, a new method (*Method III*) is introduced to estimate the gas holdup in a cocurrent bubble column. This method considers an estimation of the wall shear stress effect without modeling the two-phase wall shear stress or solving a non-linear form of Eq. (4). The procedure is as simple as Method II but provides more accurate gas holdup values.

2. Method III – a new differential pressure gas holdup estimation method

Consider rewriting Eq. (5) in the form

$$\bar{\varepsilon}_{III} = 1 - \frac{\Delta p}{\Delta p_{0,U_l}} \tag{6}$$

where $\Delta p_{0,U_l}$ is the pressure difference between z_1 and z_2 (the same locations corresponding to Δp) when $U_g = 0$ ($\bar{\varepsilon} = 0$) and U_l is the same superficial liquid velocity at which Δp is measured. Eq. (6) becomes

Eq. (5) when $U_l = 0$. The gas holdup measurement based on Eq. (6) is called *Method III* in the following discussion and $\bar{\varepsilon}_{III}$ denotes the gas holdup value obtained with Method III.

For single-phase liquid flows (i.e., $\bar{\varepsilon} = 0$), using Eq. (4), we have

$$0 = \left(1 - \frac{1}{\rho_l g}\frac{\Delta p_{0,U_l}}{\Delta z}\right) + \frac{4\bar{\tau}_{w0}}{\rho_l D_c g} \tag{7}$$

Hence,

$$\Delta p_{0,U_l} = \rho_l g \Delta z + \frac{4\bar{\tau}_{w0}}{D_c}\Delta z \tag{8}$$

where $\bar{\tau}_{w0}$ is the wall shear stress for single-phase liquid flow with the same superficial liquid velocity U_l corresponding to Δp.

Substituting Eq. (8) into (6)

$$\bar{\varepsilon}_{III} = 1 - \frac{1}{\rho_l g}\frac{\Delta p}{\Delta z}\left(\frac{1}{1 + \frac{4\bar{\tau}_{w0}}{\rho_l g D_c}}\right) \tag{9}$$

Eq. (9) reduces to Method II (Eq. (5)) as $\frac{4\bar{\tau}_{w0}}{\rho_l g D_c} \to 0$, i.e., Methods III and II are the same when measuring gas holdup in a semi-batch bubble column.

The single-phase flow wall shear stress can be estimated by

$$\bar{\tau}_{w0} = \frac{1}{2} C_f \rho_l U_l^2 \tag{10}$$

where

$$C_f = \frac{1}{4} f \tag{11}$$

For Newtonian fluid flows, the following explicit formula can be used to estimate f (Streeter and Wylie, 1985)

$$f = \frac{1.325}{\left[\ln\left(\frac{\Delta}{3.7D_c} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \tag{12}$$

where $\frac{\Delta}{D_c}$ is the relative wall roughness and Re is the liquid flow Reynolds number based on column diameter D_c.

According to Eqs. (10)–(12), $\frac{4\bar{\tau}_{w0}}{\rho_l g D_c} \ll 1$ is applicable for most bubble columns, which usually have diameters at least on the order of several centimeters and operate at superficial liquid velocities lower than 1 m/s. For example, in a 15.24 cm bubble column with water only flowing at $U_l = 1$ m/s and $\frac{\Delta}{D_c} = 0.01$, $\frac{4\bar{\tau}_{w0}}{\rho_l g D_c} = 0.013$; when $D_c = 2.54$ cm, for the same conditions $\frac{4\bar{\tau}_{w0}}{\rho_l g D_c} = 0.081$. With the $\bar{\tau}_{w0}$ model provided by Metkin and Sokolov (1982) for non-Newtonian power-law fluids, $\frac{4\bar{\tau}_{w0}}{\rho_l g D_c} \ll 1$ also holds for most bubble column conditions.

When $\frac{4\bar{\tau}_{w0}}{\rho_l g D_c} \ll 1$, a Taylor series expansion can be used to estimate Eq. (9), thus

$$\bar{\varepsilon}_{III} \approx 1 - \frac{1}{\rho_l g}\frac{\Delta p}{\Delta z} + \frac{1}{\rho_l g}\frac{\Delta p}{\Delta z}\frac{4\bar{\tau}_{w0}}{\rho_l g D_c} \tag{13}$$

Assuming the gas holdup given by Method I is accurate, then the error of Method II ($\Delta\bar{\varepsilon}_{II}$) is

$$\Delta\bar{\varepsilon}_{II} = |\bar{\varepsilon}_I - \bar{\varepsilon}_{II}| = \bar{\varepsilon}_\tau = \frac{4\bar{\tau}_w}{\rho_l D_c g} \tag{14}$$

where $\bar{\varepsilon}_\tau$ represents the contribution of wall shear stress to the total gas holdup. The error of Method III ($\Delta\bar{\varepsilon}_{\mathrm{III}}$) is

$$\Delta\bar{\varepsilon}_{\mathrm{III}} = |\bar{\varepsilon}_{\mathrm{I}} - \bar{\varepsilon}_{\mathrm{III}}| = \frac{4\bar{\tau}_{\mathrm{w}}}{\rho_{\mathrm{l}} D_{\mathrm{c}} g}\left|1 - \frac{1}{\rho_{\mathrm{l}} g}\frac{\Delta p}{\Delta z}\frac{\bar{\tau}_{\mathrm{w0}}}{\bar{\tau}_{\mathrm{w}}}\right| \tag{15}$$

Combining Eqs. (4), (14), and (15) yields

$$\frac{\Delta\bar{\varepsilon}_{\mathrm{III}}}{\Delta\bar{\varepsilon}_{\mathrm{II}}} = \left|1 - (1 - \bar{\varepsilon}_{\mathrm{I}} - \bar{\varepsilon}_\tau)\frac{\bar{\tau}_{\mathrm{w0}}}{\bar{\tau}_{\mathrm{w}}}\right| \tag{16}$$

Since $\frac{\bar{\tau}_{\mathrm{w}}}{\bar{\tau}_{\mathrm{w0}}} > 1$ (Herringe and Davis, 1978; Metkin and Sokolov, 1982; Marie, 1987),

$$\frac{\Delta\bar{\varepsilon}_{\mathrm{III}}}{\Delta\bar{\varepsilon}_{\mathrm{II}}} < 1 \tag{17}$$

This shows that the error of Method III is always smaller than that of Method II, which is an important advantage of Method III.

In air–water cocurrent upward flows, Herringe and Davis (1978) found

$$\frac{\bar{\tau}_{\mathrm{w}}}{\bar{\tau}_{\mathrm{w0}}} = 1 + 0.22\bar{\varepsilon} + 0.82\bar{\varepsilon}^2 \tag{18}$$

Using $\bar{\varepsilon}_{\mathrm{I}}$ in Eq. (18) and substituting Eq. (18) into Eq. (16)

$$\frac{\Delta\bar{\varepsilon}_{\mathrm{III}}}{\Delta\bar{\varepsilon}_{\mathrm{II}}} \approx 1 - (1 - \bar{\varepsilon}_{\mathrm{I}})\frac{1}{1 + 0.22\bar{\varepsilon}_{\mathrm{I}} + 0.82\bar{\varepsilon}_{\mathrm{I}}^2} \tag{19}$$

Using a Taylor series expansion on Eq. (19)

$$\frac{\Delta\bar{\varepsilon}_{\mathrm{III}}}{\Delta\bar{\varepsilon}_{\mathrm{II}}} \approx 1 - (1 - \bar{\varepsilon}_{\mathrm{I}})(1 - 0.22\bar{\varepsilon}_{\mathrm{I}} - 0.82\bar{\varepsilon}_{\mathrm{I}}^2) \tag{20}$$

Thus

$$\frac{\Delta\bar{\varepsilon}_{\mathrm{III}}}{\Delta\bar{\varepsilon}_{\mathrm{II}}} \approx 1.22\bar{\varepsilon}_{\mathrm{I}} \tag{21}$$

Since $\bar{\varepsilon}_{\mathrm{I}}$ is on the order of 0.1 for many bubble column operational conditions, this implies the error of Method III can be an order of magnitude lower than that of Method II.

When the liquid phase is a power-law fluid, i.e.,

$$\mu_{\mathrm{l}} = K\gamma^{n-1} \tag{22}$$

where μ_{l} is the apparent viscosity, K is the fluid consistency index, and n is the power-law index, Metkin and Sokolov (1982) recommended

$$\frac{\bar{\tau}_{\mathrm{w}}}{\bar{\tau}_{\mathrm{w0}}} = 1 + 2.4n\left(\frac{U_{\mathrm{g}}}{U_{\mathrm{l}}}\right)^{1/2} Re_n^{-0.0625n} \tag{23}$$

where

$$Re_n = \frac{8U_{\mathrm{l}}^{2-n}D^n\rho_{\mathrm{l}}}{K\left(\frac{6n+2}{n}\right)^n} \tag{24}$$

then

$$\frac{\Delta\bar{\varepsilon}_{\mathrm{III}}}{\Delta\bar{\varepsilon}_{\mathrm{II}}} \approx 1 - \frac{1}{1 + 2.4n\left(\frac{U_{\mathrm{g}}}{U_{\mathrm{l}}}\right)^{1/2} Re_n^{-0.0625n}} \tag{25}$$

Fig. 1 shows an example of the relationship between $\frac{\Delta\bar{\varepsilon}_{\mathrm{III}}}{\Delta\bar{\varepsilon}_{\mathrm{II}}}$, $\frac{U_{\mathrm{g}}}{U_{\mathrm{l}}}$, and Re_n for a 1.01% carboxymethyl cellulose solution, whose consistency index is 0.709 (Al-Masry, 2001). The value $\frac{\Delta\bar{\varepsilon}_{\mathrm{III}}}{\Delta\bar{\varepsilon}_{\mathrm{II}}}$ is always smaller than 1 and decreases with decreasing $\frac{U_{\mathrm{g}}}{U_{\mathrm{l}}}$ and increasing Re_n. When $\frac{U_{\mathrm{g}}}{U_{\mathrm{l}}} \approx 1$, $\Delta\bar{\varepsilon}_{\mathrm{III}}$ is ~50–60% of $\Delta\bar{\varepsilon}_{\mathrm{II}}$. When $\frac{U_{\mathrm{g}}}{U_{\mathrm{l}}} \approx 0.1$,

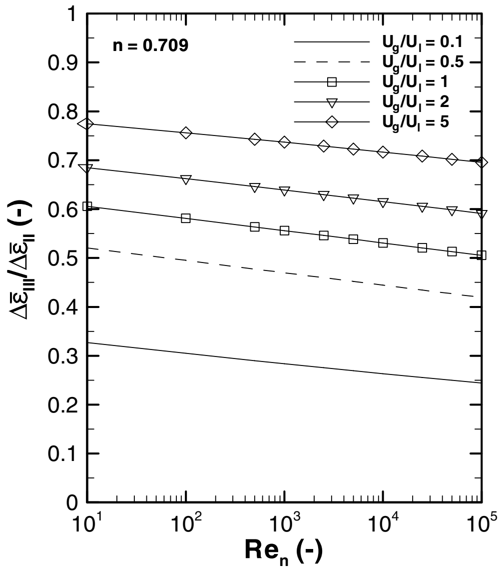

Fig. 1. Variation of $\frac{\Delta\bar{\varepsilon}_{III}}{\Delta\bar{\varepsilon}_{II}}$ with Re_n and U_g/U_l for power-law fluids ($n = 0.709$).

$\Delta\bar{\varepsilon}_{III}$ is $\sim$30% of $\Delta\bar{\varepsilon}_{II}$. The $\frac{\Delta\bar{\varepsilon}_{III}}{\Delta\bar{\varepsilon}_{II}}$ trend with $\frac{U_g}{U_l}$ and Re_n implies Method III is especially useful for flows at high superficial liquid velocity and low superficial gas velocity when total gas holdup ($\bar{\varepsilon}$) is low but $\bar{\varepsilon}_\tau$ comprises a significant part of $\bar{\varepsilon}$. Omitting $\bar{\varepsilon}_\tau$ in this case results in a substantial relative error, whereas with Method III, the error is reduced considerably.

3. Applying method III to determine gas holdup in a cocurrent air–water–fiber bubble column

In this section, Methods I–III are used to determine the gas holdup in a cylindrical cocurrent air–water–fiber bubble column at various operational conditions. The gas holdup values from the three methods are compared and the advantages of Method III over Method II are demonstrated.

The experimental system is described in detail in Tang (2005) and will be briefly introduced here. The bubble column consists of four 0.914 m tall acrylic tubes with 15.24 cm internal diameter. Five delrin collars, each 5.1 cm tall, and 11 buna-n gaskets are used to connect the acrylic tubes for a total column height of 4 m. Fig. 2 shows a schematic of the entire system. Filtered air is supplied by a compressor and enters the bubble column from the bottom via a spider sparger. The air flowrate is adjusted with a regulator and measured with one of three gas flowmeters, each covering a different flowrate range. The fiber suspension from a 379 l reservoir is pumped into the column. The pump is connected to the reservoir with a 2.44 m long 7.62 cm diameter PVC pipe. A 2.85 m long 2.54 cm diameter PVC pipe connects the pump to the column. The fiber suspension flowrate is measured with a magnetic flowmeter and varied via a pump power frequency controller. The fiber suspension enters the column through a flow expander to provide a nearly uniform liquid velocity field at the entrance region prior to the spider sparger. A gas–liquid separator is located on top of the column where air is separated from the water while the water returns to the reservoir through a PVC pipe. Along the column, 5 pressure transducers (P1, P2, P3, P4, and P5 in Fig. 2) are installed, one in each of the five delrin collars. Each acrylic tube section is numbered 1–4 from the bottom of the column. All pressure and flowmeter signals are collected via a computer controlled data acquisition system. Superficial gas and liquid velocities are controlled by a gas regulator and pump power frequency controller, respectively.

The spider sparger, shown in Fig. 3, has eight arms made of 12.7 mm diameter stainless steel tubes. Thirty-three 1.6 mm diameter holes are located on one side of each arm and distributed as shown in Fig. 3. The arms are soldered to the center cylinder of the sparger such that all the holes face the same direction. Air enters the

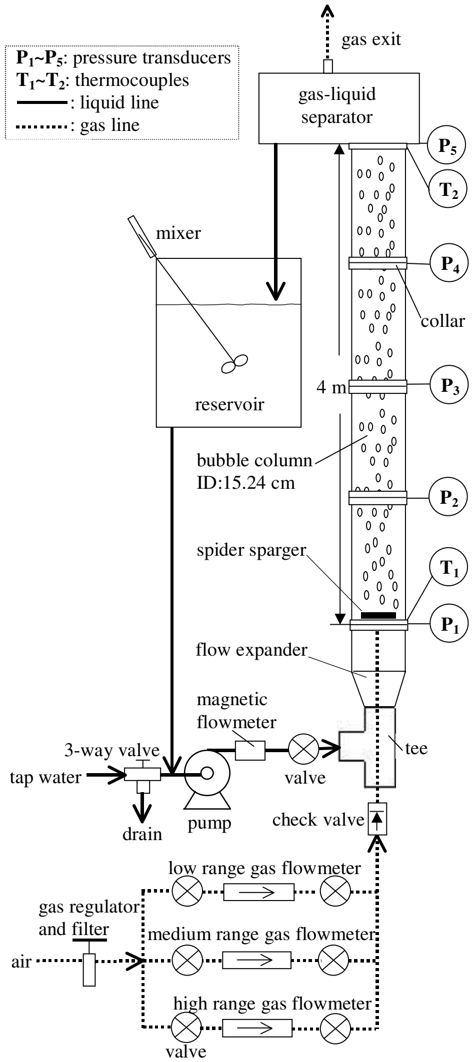

Fig. 2. Schematic of the cocurrent bubble column experimental facility.

spider sparger from the central cylinder and exits from the arm holes. The sparger is installed with the holes facing upward.

All experiments in this study are carried out under atmospheric pressure and ambient temperature. The superficial gas velocity range is $0 < U_g < 20$ cm/s, and the superficial liquid velocity range is $0 < U_l < 10$ cm/s. Eucalyptus wood fiber and tap water comprise the fiber suspension. The fibers have a length-weighted average fiber length of ~0.8 mm and a fiber coarseness index of ~7.2 mg/100 m. All fiber is disintegrated from dry lap fiber sheets. The fiber sheets are originally torn into small pieces and then a specified mass of oven-dry fiber is weighed. It is then soaked in tap water for 24 h before the pieces of fiber sheet are disintegrated in a Black–Clawson laboratory hydropulper. The concentrated fiber suspension is then transferred to the reservoir and additional tap water is added to raise the suspension to a predetermined level. Fiber mass fraction C is defined as the ratio of the oven-dry fiber mass to the suspension mass.

To acquire gas holdup data at a given U_g and U_l, 4800 readings are collected from each instrument every 10 ms and averaged after quasi-steady conditions are reached. The pressure transducer (Cole–Parmer, Model: 68075) error is less than 0.25% of full scale (34.5 kPa) for a single measurement. When pressure signal fluctuations are significant due to large bubble passage, the variation between two successive measurements is large. However, with multiple (e.g., 4800) measurements, the resultant average pressure is much more precise. For

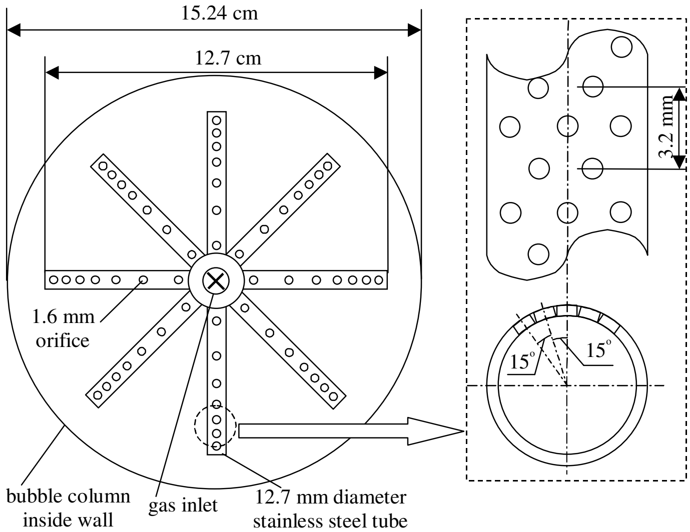

Fig. 3. Schematic of the spider sparger.

example, for an average pressure of 4800 measurements, the standard deviation of the average pressure is only ~1/70 of that of a single measurement (Figliola and Beasley, 2000). Hence, the error in the pressure measurements should be much smaller than 0.25% of full scale. In the present study, the pressure difference used $(p_1 - p_4)$ is ~20–30 kPa, which is on the same order of magnitude as the pressure transducer full scale. Hence, the relative error of pressure difference should be smaller than 0.25%.

With five pressure signals, the averaged (both temporal and spatial) gas holdup in each section and the overall column gas holdup (the average gas holdup in Sections 1–3) can be calculated.

The pressure drop per unit length $\left(\frac{\Delta p_\tau}{\Delta z}\right)$ due to wall shear stress in a fiber suspension flowing at a given superficial liquid velocity (U_l) without aeration is estimated by

$$\frac{\Delta p_\tau}{\Delta z} = \frac{\Delta p_{0,U_l} - \Delta p_0}{\Delta z} \tag{26}$$

where $\Delta p_0 = \rho_l g \Delta z$, i.e., the hydrostatic head of a static fluid column.

The wall shear stress for a fiber suspension flow when $U_g = 0$ is calculated by

$$\bar{\tau}_{w0} = \frac{D_c \Delta p_\tau}{4\Delta z} = \frac{D_c(\Delta p_{0,U_l} - \Delta p_0)}{4\Delta z} \tag{27}$$

No direct wall shear stress correlation has been found for gas–fiber suspension cocurrent flows. Note that when $C \leqslant 1.5\%$, the fiber suspension is considered dilute and it behaves like a Newtonian fluid in a fluidized state (Seely, 1968; Gullichsen and Harkonen, 1981; Bennington and Kerekes, 1996) and its apparent viscosity can be estimated as $\mu_a = 1.5 \times 10^{-3} C^{3.1}$ Pa s when $1\% < C \leqslant 12.6\%$ (Kerekes, 1996). In the present bubble column, because bubbles acts as "mobile mixers" (Heindel and Garner, 1999), the dilute fiber suspensions are assumed to be fluidized. To determine the average gas holdup in an air–water–fiber suspension for fiber mass fractions $C \leqslant 1.5\%$ with Method I, assume the Herringe and Davis (1978) correlation (i.e., Eq. (18)) for wall shear stress in two-phase flow is applicable. Hence, the average gas holdup via Method I is

$$\bar{\varepsilon}_I = 1 - \frac{\Delta p}{\Delta p_0} + \frac{\Delta p_{0,U_l} - \Delta p_0}{\Delta p_0}(1 + 0.22\bar{\varepsilon}_I + 0.82\bar{\varepsilon}_I^2) \tag{28}$$

It is acknowledged that the gas holdup obtained by Method I is affected by the specific wall shear stress correlation and the selection of a suitable and accurate wall shear stress correlation is very critical for obtaining an accurate gas holdup measurement with Method I. However, the application of Eq. (18) does not affect the measurement accuracy of Methods II and III because they do not need a wall shear stress correlation to

Table 1
Comparison between Methods I–III at selected operating conditions in a cocurrent air–water bubble column ($C = 0\%$) when the nominal superficial gas velocity is 20 cm/s

U_l (cm/s)	U_g (cm/s)	$\bar{\varepsilon}_I$ (Eq. (4))	$\bar{\varepsilon}_{II}$ (Eq. (5))	$\bar{\varepsilon}_{III}$ (Eq. (6))	$\frac{\Delta\bar{\varepsilon}_{II}}{\bar{\varepsilon}_I}$[a] (%)	$\frac{\Delta\bar{\varepsilon}_{III}}{\bar{\varepsilon}_I}$[a] (%)	$\frac{\Delta\varepsilon_{III}}{\Delta\bar{\varepsilon}_{II}}$[a] (%)
0.1	20.9	0.230	0.230	0.230	0	0	NA
2.1	20.5	0.219	0.219	0.219	0.08	0.02	22.0
4.0	20.5	0.215	0.214	0.215	0.19	0.04	21.6
6.0	20.8	0.213	0.212	0.212	0.21	0.05	21.4
8.2	21.3	0.209	0.208	0.208	0.26	0.06	21.0
10.0	20.8	0.203	0.202	0.202	0.31	0.06	20.4

[a] $\frac{\Delta\bar{\varepsilon}_{II}}{\bar{\varepsilon}_I}$, $\frac{\Delta\bar{\varepsilon}_{III}}{\bar{\varepsilon}_I}$, and $\frac{\Delta\varepsilon_{III}}{\Delta\bar{\varepsilon}_{II}}$ are calculated with unrounded $\bar{\varepsilon}_I$, $\bar{\varepsilon}_{II}$, and $\bar{\varepsilon}_{III}$ values.

Table 2
Comparison between Methods I–III at selected operating conditions in an air–water–fiber bubble column when $C = 1.5\%$ and the nominal superficial gas velocity is 20 cm/s

U_l (cm/s)	U_g (cm/s)	$\bar{\varepsilon}$ (Eq. (4))	$\bar{\varepsilon}_{II}$ (Eq. (5))	$\bar{\varepsilon}_{III}$ (Eq. (6))	$\frac{\Delta\bar{\varepsilon}_{II}}{\bar{\varepsilon}}$[a] (%)	$\frac{\Delta\bar{\varepsilon}_{III}}{\bar{\varepsilon}}$[a] (%)	$\frac{\Delta\varepsilon_{III}}{\Delta\bar{\varepsilon}_{II}}$[a] (%)
0.1	18.2	0.154	0.154	0.154	0	0	NA
2.0	20.8	0.166	0.163	0.166	1.63	0.27	16.6
3.9	20.3	0.161	0.156	0.160	2.66	0.43	16.1
6.0	20.5	0.160	0.153	0.159	4.15	0.66	15.9
8.0	20.2	0.155	0.148	0.154	4.89	0.75	15.4
10.1	20.4	0.155	0.147	0.154	5.28	0.81	15.4

[a] $\frac{\Delta\bar{\varepsilon}_{II}}{\bar{\varepsilon}_I}$, $\frac{\Delta\bar{\varepsilon}_{III}}{\bar{\varepsilon}_I}$, $\frac{\Delta\varepsilon_{III}}{\Delta\bar{\varepsilon}_{II}}$ are calculated with unrounded $\bar{\varepsilon}_I$, $\bar{\varepsilon}_{II}$, and $\bar{\varepsilon}_{III}$ values.

calculate gas holdup. Instead, Method III provides an alternate way to account for the wall shear friction effect on gas holdup measurements. The results from Method I are used as a reference, the choice of Eq. (18) does not affect the relationship between Methods II and III.

By measuring Δp_0, $\Delta p_{0,U_l}$, and Δp for different operational conditions, accurate gas holdup values ($\bar{\varepsilon}_I$) can be obtained by solving Eq. (28). The gas holdup can be estimated using Method III (Eq. (6)), where no assumptions are needed with respect to a wall shear stress model for two-phase flows. The gas holdup obtained by Method II (Eq. (5)) can be written as

$$\bar{\varepsilon}_{II} = 1 - \frac{\Delta p}{\Delta p_0} \tag{29}$$

The error associated with Method II is

$$\Delta\bar{\varepsilon}_{II} = \bar{\varepsilon}_\tau = \frac{\Delta p_{0,U_l} - \Delta p_0}{\Delta p_0}(1 + 0.22\bar{\varepsilon}_I + 0.82\bar{\varepsilon}_I^2) \tag{30}$$

Tables 1 and 2 compare overall column gas holdup values obtained using Methods I–III and errors associated with Methods II and III at a fixed nominal superficial gas velocity ($U_g = 20$ cm/s) in air–water and air–water–fiber ($C = 1.5\%$) systems. Note the gas holdup values ($\bar{\varepsilon}_I$, $\bar{\varepsilon}_{II}$, and $\bar{\varepsilon}_{III}$) presented in these two tables are rounded to 0.001 while the relative errors of Methods II and III $\left(\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I} \text{ and } \frac{\Delta\bar{\varepsilon}_{III}}{\bar{\varepsilon}_I}\right)$ are calculated using the unrounded gas holdup values. The overall column gas holdup values are calculated with the pressure values measured with transducer P1 and P4 (Fig. 2). Although the relative errors are small in Table 1, the consistent trends (i.e., $\frac{\Delta\bar{\varepsilon}_{II}}{\bar{\varepsilon}} > \frac{\Delta\bar{\varepsilon}_{III}}{\bar{\varepsilon}}$ when $U_l > 0$, and $\frac{\Delta\bar{\varepsilon}_{II}}{\bar{\varepsilon}}$ and $\frac{\Delta\bar{\varepsilon}_{III}}{\bar{\varepsilon}}$ increase with increasing U_l) shown in Table 1 indicate that the pressure difference measurement is sufficiently accurate, and the gas holdup values from Methods II and III can be reliably differentiated.

In air–water systems (Table 1), both the errors resulted from Methods II and III are negligible because the superficial liquid velocity is small ($0 \leqslant U_l \leqslant 10$ cm/s). However, the error resulting from Method III is an order of magnitude lower than that from Method II. In the air–water–fiber systems at $C = 1.5\%$ (Table 2), the errors resulting from both Methods II and III increase by an order of magnitude and the relative error for Method II increases to ~5% of the total gas holdup. However, the error resulting from Method III is still lower than 1% and is only ~15% of that from Method II.

The variation of overall column gas holdup values ($\bar{\varepsilon}_{\mathrm{I}}$, $\bar{\varepsilon}_{\mathrm{II}}$, and $\bar{\varepsilon}_{\mathrm{III}}$) from the three methods at different superficial gas velocities when $U_{\mathrm{l}} = 10$ cm/s and $C = 1.5\%$ are compared in Fig. 4. The gas holdup values from Method III are almost the same as those from Method I for all addressed superficial gas velocities while $\bar{\varepsilon}_{\mathrm{II}}$ always deviates from $\bar{\varepsilon}_{\mathrm{I}}$ by ~0.01. The relative errors $\left(\frac{\Delta\varepsilon_{\mathrm{II}}}{\bar{\varepsilon}_{\mathrm{I}}}\right.$ and $\left.\frac{\Delta\varepsilon_{\mathrm{III}}}{\bar{\varepsilon}_{\mathrm{I}}}\right)$ associated with Methods II and III for the same conditions are presented in Fig. 5. The relative error of Method III $\left(\frac{\Delta\varepsilon_{\mathrm{III}}}{\bar{\varepsilon}_{\mathrm{I}}}\right)$ is always lower than

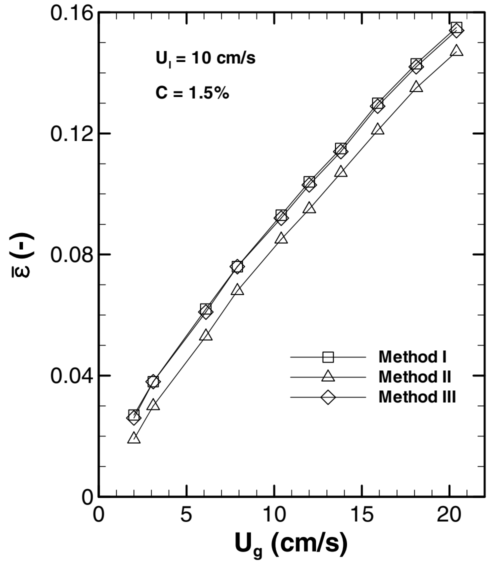

Fig. 4. Comparison between gas holdup values from Methods I to III at $0 < U_{\mathrm{g}} < 20$ cm/s when $U_{\mathrm{l}} = 10$ cm/s and $C = 1.5\%$.

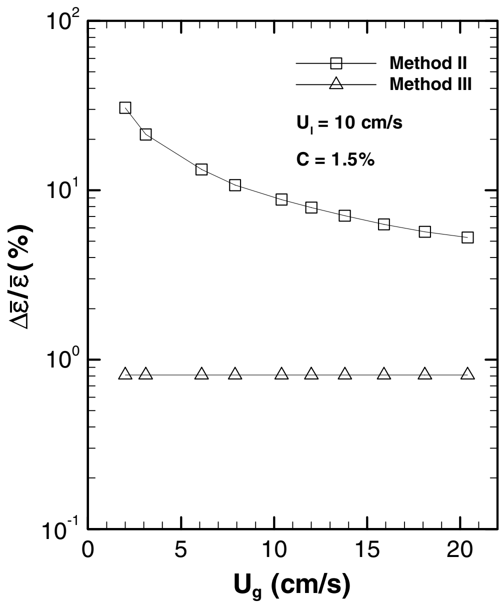

Fig. 5. Comparison between relative gas holdup error from Methods II and III at $0 < U_{\mathrm{g}} < 20$ cm/s when $U_{\mathrm{l}} = 10$ cm/s and $C = 1.5\%$.

1% and nearly constant in this superficial gas velocity range, while the relative error of Method II $\left(\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}\right)$ is much higher, ranging from ~5% at $U_g \approx 20$ cm/s to ~30% at $U_g \sim 2.0$ cm/s. Hence, wall shear effects are significant at high U_l and low U_g, and if they are not properly accounted for in Method II, they can produce a significant error in gas holdup measurements.

Since results at $C = 1.5\%$ (Table 2 and Figs. 4 and 5) show that Method III results in a negligible error in gas holdup measurements when $0 \leqslant U_l \leqslant 10$ cm/s and $0 \leqslant U_g \leqslant 20$ cm/s, the error of Method III is also negligible at $C < 1.5\%$ since the wall shear stress at $C < 1.5\%$ will be lower than that at $C = 1.5\%$ in the same superficial liquid velocity range (Forgacs et al., 1958).

Due to the pump capacity and the air supply used in the experimental system, gas holdup data with Methods I–III can only be obtained at $U_g \leqslant 20$ cm/s and $U_l \leqslant 10$ cm/s. However, the performance of Methods II and III (i.e., $\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}$ and $\frac{\Delta\varepsilon_{III}}{\bar{\varepsilon}_I}$) at higher U_g and U_l can still be compared. For an air–water–fiber cocurrent bubble column, using Eqs. (14) and (18) and substituting $\bar{\varepsilon}$ with $\bar{\varepsilon}_I$ results in

$$\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I} \approx \frac{4\bar{\tau}_{w0}}{\rho_l D_c g}\left(\frac{1}{\bar{\varepsilon}_I} + 0.22\right) \tag{31}$$

Combining Eqs. (31) and (21) gives

$$\frac{\Delta\varepsilon_{III}}{\bar{\varepsilon}_I} \approx \frac{4\bar{\tau}_{w0}}{\rho_l D_c g}(1.22 + 0.27\bar{\varepsilon}_I) \tag{32}$$

Since U_l only slightly affects $\bar{\varepsilon}_I$ (Tang and Heindel, 2005a,b), the effect of U_l on $\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}$ and $\frac{\Delta\varepsilon_{III}}{\bar{\varepsilon}_I}$ is mainly due to its influence on $\bar{\tau}_{w0}$, hence, it is expected that both $\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}$ and $\frac{\Delta\varepsilon_{III}}{\bar{\varepsilon}_I}$ increase with increasing U_l since $\bar{\tau}_{w0}$ increases. However, the ratio $\frac{\Delta\bar{\varepsilon}_{III}}{\Delta\bar{\varepsilon}_{II}}$ will not change significantly because it is only a function of $\bar{\varepsilon}_I$ (Eq. (21)). The effect of U_g on $\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}$ and $\frac{\Delta\varepsilon_{III}}{\bar{\varepsilon}_I}$ can be seen from its influence on $\bar{\varepsilon}_I$. Since $\bar{\varepsilon}_I$ increases with increasing U_g, $\frac{\Delta\varepsilon_{III}}{\bar{\varepsilon}_I}$ increases while $\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}$ decreases with increasing U_g. However, because $\bar{\varepsilon}_I$ is always smaller than 1 and in most cases smaller than 0.5, $\frac{\Delta\varepsilon_{III}}{\bar{\varepsilon}_I}$ only changes slightly in a small range (Eq. (32)) even at a very high U_g. $\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}$ is very large if $\bar{\varepsilon}_I$ is very small, which has been shown in Fig. 5. As U_g increases, $\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}$ will asymptotically approach a lower limit. Because $\bar{\varepsilon}_I$ is usually smaller than 0.5, $\frac{\Delta\varepsilon_{II}}{\bar{\varepsilon}_I}$ is still significantly larger than $\frac{\Delta\varepsilon_{III}}{\bar{\varepsilon}_I}$, even at a high U_g.

It is noted that in some cases Method II is sufficiently accurate (e.g., the conditions in Table 1). However, in other cases (e.g., the conditions in Table 2 and Fig. 5), Method II can cause significant errors. Method III always results in more accurate gas holdup measurements than Method II. Its accuracy is much more consistent than that of Method II (Fig. 5) and is sufficiently high for all the conditions presented in Tables 1 and 2 and other conditions not presented (Tang, 2005). Furthermore, the only additional work when Method III is used instead of Method II is to measure the single-phase liquid flow pressure drop at each investigated superficial liquid velocity using the same cocurrent bubble column and transducers. Therefore, it is worthwhile to use Method III instead of Method II in measuring gas holdup in bubble columns whenever possible.

4. Conclusion

A new gas holdup estimation method (Method III) via differential pressure measurements for cocurrent bubble columns was proposed. This method considers the wall shear stress influences on gas holdup values by modifying Eq. (5) to produce Eq. (6). A detailed analysis revealed that Method III always results in a smaller gas holdup error than Method II. In many cases, the error is much smaller than that of Method II. Hence, using Method III, more accurate gas holdup measurements in cocurrent bubble columns can be made with only pressure measurements, and the calculation is as simple as that required by Method II. Furthermore, no knowledge of wall shear stress is required for Method III, which is not the case for Method I. The applicability of Method III in the present study to gas holdup in a cocurrent air–water–fiber bubble column was examined. Analysis based on experimental data showed that with Method III, accurate gas holdup values can be obtained consistently, while error may be significant for selected operational conditions with Method II.

Acknowledgments

Portions of this project were supported by the National Research Initiative of the USDA Cooperative State Research, Education and Extension Service, grant number 2001-35103-11259. The cellulose fiber supply and characterization was provided by Kimberly-Clark Corporation; their support is appreciated.

Appendix A. Discussion on the applicability of Methods I–III to calculate two-phase wall shear stress

Among the equations for Methods I–III, Eqs. (4)–(6), only Eq. (4) (i.e., the principle of Method I) includes the wall shear stress term, then Eq. (4) must be used if wall shear stress is to be calculated with Methods I–III.

With the pressure difference between two axial locations separated by a distance $\Delta z = z_2 - z_1$ along the bubble column, Eq. (4) can be used to solve for the average wall shear stress between these two locations providing the average gas holdup between the same locations can be accurately measured. This requires an accurate gas holdup measurement other than Method I.

There are many gas holdup measurement methods as reviewed by Kumar et al. (1997) and Boyer et al. (2002). Certainly, some of those methods (e.g., the dynamic gas disengagement method), if properly arranged, can provide the gas holdup required to calculate the wall shear stress with Eq. (4). The details are not addressed here. In the following, we focus on the possibility to estimate the average wall shear stress with only pressure difference measurements, i.e., Methods I–III.

If only Methods I–III are available, the two possible ways to measure the average two-phase wall shear stress $\bar{\tau}_w$ are to approximate $\bar{\varepsilon}_I$ in Eq. (4) with $\bar{\varepsilon}_{II}$ or $\bar{\varepsilon}_{III}$ and then calculate an approximate value (denoted as $\tilde{\tau}_w$ in the following) of the average wall shear stress.

If $\bar{\varepsilon}_I$ is approximated with $\bar{\varepsilon}_{II}$, according to Eqs. (4) and (5), the approximate wall shear stress $\tilde{\tau}_w$ will always be zero. This is not acceptable.

If $\bar{\varepsilon}_I$ is approximated with $\bar{\varepsilon}_{III}$, according to Eqs. (4) and (5),

$$\frac{4\bar{\tau}_w}{\rho_l g D_c} = \bar{\varepsilon}_I - \bar{\varepsilon}_{II} \tag{33}$$

Here we set

$$\frac{4\tilde{\tau}_w}{\rho_l g D_c} = \bar{\varepsilon}_{III} - \bar{\varepsilon}_{II} \tag{34}$$

Substituting Eqs. (5) and (9) in Eq. (34) yields

$$\frac{4\tilde{\tau}_w}{\rho_l g D_c} = \frac{\Delta p}{\Delta p_0} \frac{\frac{4\bar{\tau}_{w0}}{\rho_l g D_c}}{1 + \frac{4\bar{\tau}_{w0}}{\rho_l g D_c}} \tag{35}$$

i.e.,

$$\frac{\tilde{\tau}_w}{\bar{\tau}_{w0}} = \frac{\Delta p}{\Delta p_0} \frac{1}{1 + \frac{4\bar{\tau}_{w0}}{\rho_l g D_c}} = \frac{\Delta p}{\Delta p_{0,U_l}} = 1 - \bar{\varepsilon}_{III} \leqslant 1 \tag{36}$$

Thus, $\tilde{\tau}_w$ is only close to the accurate value $\bar{\tau}_w$ when the gas holdup is very small, i.e., when $\bar{\tau}_w$ is close to $\bar{\tau}_{w0}$. When gas holdup is large, $\tilde{\tau}_w$ cannot represent $\bar{\tau}_w$ because $\frac{\tilde{\tau}_w}{\bar{\tau}_{w0}} \leqslant 1$, while $\frac{\bar{\tau}_w}{\bar{\tau}_{w0}} > 1$ according to literature (Herringe and Davis, 1978; Metkin and Sokolov, 1982; Marie, 1987).

Furthermore, according to Eq. (18), $\frac{\bar{\tau}_w}{\bar{\tau}_{w0}}$ increases with increasing gas holdup, or according to Eq. (23), $\frac{\bar{\tau}_w}{\bar{\tau}_{w0}}$ increases with increasing superficial gas velocity, which usually leads to higher gas holdup when other conditions are the same. However, according to Eq. (36), $\frac{\tilde{\tau}_w}{\bar{\tau}_{w0}}$ decreases with increasing gas holdup or superficial gas velocity.

Hence, $\tilde{\tau}_w$ cannot be used to approximate $\bar{\tau}_w$ for most conditions even when $\bar{\varepsilon}_{III}$ is used to approximate the gas holdup value. This is expected as Method III only includes a portion of the wall shear stress contribution when calculating gas holdup and this portion is larger when gas holdup is smaller.

In summary, two-phase wall shear stress cannot be accurately measured from only pressure difference measurements (i.e., Methods I–III) at most conditions. However, the value of Method III should not be underestimated, because it provides more accurate gas holdup measurements than Method II while keeping the procedure simple. It also provides accurate gas holdup measurements at conditions where the wall shear stress term (Eq. (4)) composes a significant part of the total gas holdup, while Method II may generate unacceptable large errors.

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